

1-1. Find the transformation matrix that rotates the axis  $x_3$  of a rectangular coordinate system  $45^\circ$  toward  $x_1$  around the  $x_2$ -axis.

1-3. Find the transformation matrix that rotates a rectangular coordinate system through an angle of  $120^\circ$  about an axis making equal angles with the original three coordinate axes.

1-7. Consider a unit cube with one corner at the origin and three adjacent sides lying along the three axes of a rectangular coordinate system. Find the vectors describing the diagonals of the cube. What is the angle between any pair of diagonals?

1-9. For the two vectors

$$\mathbf{A} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}, \quad \mathbf{B} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$$

find

- (a)  $\mathbf{A} - \mathbf{B}$  and  $|\mathbf{A} - \mathbf{B}|$     (b) component of  $\mathbf{B}$  along  $\mathbf{A}$     (c) angle between  $\mathbf{A}$  and  $\mathbf{B}$   
 (d)  $\mathbf{A} \times \mathbf{B}$     (e)  $(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} + \mathbf{B})$

1-10. A particle moves in a plane elliptical orbit described by the position vector

$$\mathbf{r} = 2b \sin \omega t \mathbf{i} + b \cos \omega t \mathbf{j}$$

- (a) Find  $\mathbf{v}$ ,  $\mathbf{a}$ , and the particle speed.  
 (b) What is the angle between  $\mathbf{v}$  and  $\mathbf{a}$  at time  $t = \pi/2\omega$ ?

1-20. Show that

$$(a) \sum_{ij} \epsilon_{ijk} \delta_{ij} = 0 \quad (b) \sum_{j,k} \epsilon_{ijk} \epsilon_{ijk} = 2\delta_{il} \quad (c) \sum_{i,j,k} \epsilon_{ijk} \epsilon_{ijk} = 6$$

1-22. Evaluate the sum  $\sum_k \epsilon_{ijk} \epsilon_{lmk}$  (which contains 3 terms) by considering the result for all possible combinations of  $i, j, l, m$ ; that is,

- (a)  $i = j$     (b)  $i = l$     (c)  $i = m$     (d)  $j = l$     (e)  $j = m$     (f)  $l = m$   
 (g)  $i \neq l$  or  $m$     (h)  $j \neq l$  or  $m$

Show that

$$\sum_k \epsilon_{ijk} \epsilon_{lmk} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$$

and then use this result to prove

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$